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Elastic Wave Propagation in Heterogeneous Microstructures: Applications in Diagnostic Ultrasound and Biomedical Assessment

Dr. E. Morales[1], Dr. T. Becker[1,2], Dr. S. Iqbal[2], Dr. K. Tanaka[2], Prof. D. Williams[1]*

[1] Department of Internal Medicine and Clinical Sciences, University of Melbourne, Melbourne, VIC 3010, Australia

[2] Institute of Public Health and Epidemiology, Charité – Universitätsmedizin Berlin, Berlin 10117, Germany

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Introduction

For the stress-strain apparatus introduced in section 2.1, the errors are $\pm 2.3\%$ for Young's modulus, $\pm 1.8\%$ for Poisson's ratio and ± 0.008 for Young's attenuation and Poisson's phase difference. Text S1-S2 and Figure S1-S2 show the calculation of the error. In addition, there is residual brine in the sample as the sample is saturated with oil. Therefore, there might be the mesoscopic flow – typically as patchy model- during the experiment. Text S3 and Figure 3 investigate the influence the mixture fluid on the attenuation and dispersion based on White's model incorporating Dutta-Ode correction. Tables S1 to S2 are the parameters used for the prediction.

Text S1.

Strain in the paper was obtained using the Wheatstone half-bridge consisting of two constant resistances (1000 ohms) and two strain gauges. The strain gauge factor is defined as $F = \frac{\Delta R}{R}/\varepsilon$, where ε is the strain, and ΔR is the variation of the resistance induced by the strain. The preset resistance of the strain gauge is 1000 ohms. However, in actual conditions, the resistance of the strain gauge ranges from 900 to 1100Ω due to manufacturing precision limit. The resistance bias will lead to uncertainty in the strain measurement. The output voltage of the half-bridge is:

$$U_o = \frac{R_1}{R+R_1}U_i - \frac{R}{R+R_2}U_i = \frac{R_1R_2 - R^2}{(R_1+R)(R_2+R)}U_i \approx \frac{(R_1'+R_2')R+(R_1''+R_2'')R+R_1'R_2'+R_1'R_2''+R_1''R_2'}{(R+R_1'+R)(R+R_2'+R)}U_i \quad \text{(A1)}$$

where the U_o is the output voltage, U_i is the input voltage. R_1 and R_2 represent the real resistance of the strain gauges and expressed as

$$R_1 = R + R_1' + R_1'', R_2 = R + R_2' + R_2'', \quad \text{(A2)}$$

where R_1' and R_2' represent the difference between the strain gauge #1&2 and the preset resistance R, R_1'' and R_2'' represents the variation in the resistance of the strain gauge #1&2 when the stress is applied to the sample. We have $R_1'', R_2'' \ll R$, which can be used to simplify the results.
As a consequence,
$R_{11} = R + R_1', \frac{R_{11}}{R} = \gamma_1; R_{22} = R + R_2', \frac{R_{22}}{R} = \gamma_2$, then the output voltage is

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$$U_o = \frac{(R'_1 + R'_2)R + R'_1R'_2}{(R + R_{11})(R + R_{22})}U_i + \frac{R''_1(R + R'_2)}{(R + R_{11})(R + R_{22})}U_i + \frac{R''_2(R + R'_1)}{(R + R_{11})(R + R_{22})}U_i$$
$$= \frac{\frac{R'_1 + R'_2}{R} + \frac{R'_1R'_2}{R^2}}{(1 + \gamma_1)(1 + \gamma_2)}U_i + \frac{R_{11}R_{22}}{(1 + \gamma_1)(1 + \gamma_2)}\frac{R''_1}{R_{11}}U_i + \frac{R_{11}R_{22}}{(1 + \gamma_1)(1 + \gamma_2)}\frac{R''_2}{R_{22}}U_i$$
$$= \frac{\frac{R'_1 + R'_2}{R} + \frac{R'_1R'_2}{R^2}}{(1+\gamma_1)(1+\gamma_2)}U_i + \frac{R_{11}R_{22}}{(1+\gamma_1)(1+\gamma_2)}F\varepsilon_1U_i + \frac{R_{11}R_{22}}{(1+\gamma_1)(1+\gamma_2)}F\varepsilon_2U_i \quad \text{(A3)}$$

where $\varepsilon_1, \varepsilon_2$ are the strains of two strain gauges. Basically, the strains should be the same, $\varepsilon_1 = \varepsilon_2 = \varepsilon$, then

$$U_o = \frac{\gamma_1\gamma_2 - 1}{(1+\gamma_1)(1+\gamma_2)}U_i + \frac{\gamma_1\gamma_2}{(1+\gamma_1)(1+\gamma_2)}2F\varepsilon U_i \quad \text{(A4)}$$

where the $\frac{\gamma_1\gamma_2 - 1}{(1+\gamma_1)(1+\gamma_2)}U_i$ is a constant term, which will be removed during the signal processing using the FFT filter. Then the output strain is simplified as

$$\varepsilon = \frac{U_o}{\frac{\gamma_1\gamma_2}{(1+\gamma_1)(1+\gamma_2)}2FU_i} = \frac{(1+\gamma_1)(1+\gamma_2)}{2\gamma_1\gamma_2F}\frac{U_o}{U_i} \quad \text{(A5)}$$

Therefore, the strain ε is related to the parameters $[\gamma_1, \gamma_2]$ and gauge factor F.

Based on Borgomano et al. (2017)'s , the uncertainty of the strain ε is given as,

$$\frac{\delta\varepsilon}{\varepsilon} = u(\varepsilon) = u([\gamma_1, \gamma_2]) + u(F) \quad \text{(A6)}$$

where $u([\gamma_1, \gamma_2])$ is present as,

$$u([\gamma_1, \gamma_2]) = \frac{\left[\frac{(1+\gamma_1)(1+\gamma_2)}{2\gamma_1\gamma_2} - \frac{(1+\gamma_1)(1+\gamma_2)}{2\gamma_1\gamma_2}|_{\gamma_1=\gamma_2=1}\right]}{\frac{(1+\gamma_1)(1+\gamma_2)}{2\gamma_1\gamma_2}|_{\gamma_1=\gamma_2=1}} = \frac{(1+\gamma_1)(1+\gamma_2)}{4\gamma_1\gamma_2} - 1 \quad \text{(A7)}$$

$u(F)$ is the uncertainty of the gauge factor, which is claimed to be 5% by the manufacture.

In our experiment, we carefully selected the strain gauges so that there is almost no difference between the resistance of each strain gauge: 1007 ohms and 1004 ohms for Pair 1; 1006ohms and 1005ohms for Pair 2; 1003,1004,1002 and 1005 for reference aluminum. Therefore, according to equation A7, the uncertainty $u([\gamma_1, \gamma_2])$ for Pair 1 and Pair 2 are both 0.5%. Then the uncertainty of the strain is around 5.5% for Pair #1 and Pair #2, according to equation (A6).

The Young's modulus is calculated as:

$$E = \frac{\varepsilon_{Al}}{\varepsilon_{ax}} * Ea \quad \text{(A8)}$$

Where Ea=72 GPa (Young modulus of Aluminium). The uncertainty of the Young's modulus is given as

$$u(E) = u(\varepsilon_{Al}) + u(\varepsilon_{ax}) \quad \text{(A9)}$$

Then $u(E)$=5.35%+5.5%=10.85%, which is the uncertainty of the Young's modulus in theory. Similary, the relative uncertainties in Poisson's ratio is 10.85%. These uncertainties correspond to an error of $\pm$0.95 GPa for E and $\pm$0.012 for v. Propagating the uncertainty $u(E)$ and $u(v)$ using the formula

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$$K = \frac{E}{3(1-2v)} \tag{A10}$$
$$G = \frac{E}{2(1+v)} \tag{A11}$$

We get the errors of $\pm$1.18 for K and $\pm$0.75 for G.

However, when we investigated the uncertainty of the Young's modulus using a standard sample, aluminum for example, we found the uncertainty of Young's modulus and Poisson's ratio below than 10.85%. Indeed, for the aluminum sample with the nominal value (E=72 GPa, v=0.33), the measurement is 71 GPa for Young's modulus, 0.333 for Poisson's ratio. These corresponds to the errors (uncertainty) of 1.4% for Young's modulus and 1% for Poisson's ratio, which are much smaller than the theoretical uncertainty of 10.85%. Indeed i) the uncertainty of the gauge factor F may be lower than the nominal value of 5% and the system compensates the error coming from the strain gauge. After a several measurements done on aluminum, we found an errors (uncertainties) for Young's modulus to be $\pm$2.0% (4%) and $\pm$1.5% (3%) for Poisson's ratio.

In addition, misorientation of the strain gauge can cause an additional uncertainty. We investigate the uncertainty using signals from three strain gauges at the mid-height of the aluminum sample. The standard deviation of these signals is calculated using

$$\delta x = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}} \tag{A12}$$

where the n is the number of measurements (three). $\bar{x}$ is the average of signal x_i. Figure S1a show the signals measured at three positions. The corresponding uncertainty $u(\varepsilon)$ shown in Figure S1b is around 0.3% below 100 Hz. Using the formula A9, the uncertainty of Young's modulus is around 0.6%. Similary, the relative uncertainties in Poisson's ratio is 0.6%.

Taking the uncertainty of the system (4% for E, 3% for v) into account, we expect that the uncertainty in Young's modulus is 4.0%+0.6%=4.6%, and in Poisson's ratio 3%+0.6%=3.6%. These correspond to errors of $\pm$2.3% for Young's modulus and $\pm$1.8% for Poisson's ratio. We propagate the uncertainties to bulk modulus K and shear modulus G using A10&11. This gives an error of $\pm$0.45 GPa ($\pm$4.1%) for K and $\pm$0.3 GPa ($\pm$4.1%) for G.

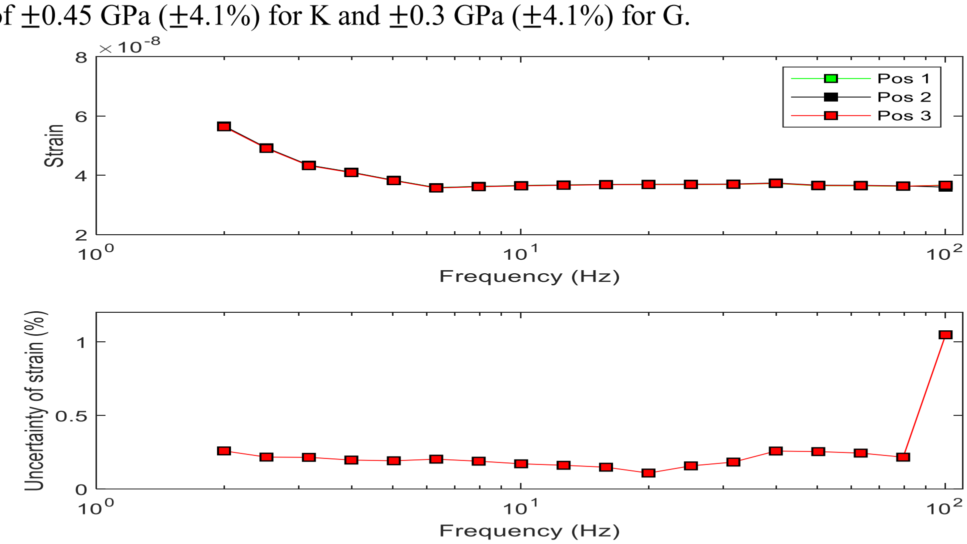

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Figure S1. The uncertainty estimation of strain measured at different locations around the sample of the aluminum. The top plot is the signal and the bottom plot is the uncertainty of the strain, which is the ratio of standard deviation to the mean of the signal.

Text S2.

In our measurement, the error of Young's attenuation is ± 0.005, which is estimated by previous measurement of the aluminum, a zero-attenuation medium. In addition, the standard deviation (Figure S2b) of the signals (Figure S2a) from different gauge is calculated using formula A10. The max value reaches 0.003 at 2 Hz. Then the max error of the Young's attenuation equals $\pm$(0.005+0.003). Propagating these uncertainties in the calculation gives the error of ± 0.012 for bulk attenuation and ± 0.012 for shear attenuation.

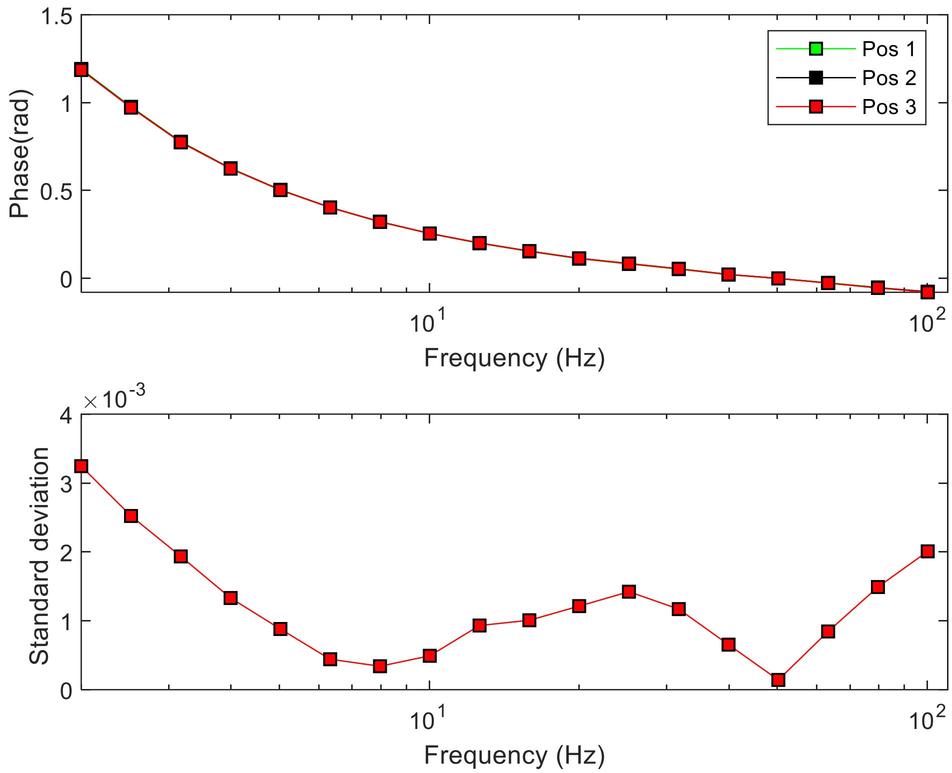

Figure S2. The uncertainty estimation of phase in signal. The top plot is the signal from different strain gauges glued at different positions on the aluminum sample. The bottom plot is the standard deviation.

Text S3.

Here we calculated the dispersion and attenuation based on the White's model incorporating Dutta-Ode correction to investigate the influence the mixture of oil and brine. The parameters used are shown in Table S1 and Table S2. The dispersion and attenuation are at a negligible level as shown in Figure S3.

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Table S1. Petro-physical properties of the sandstone. All the parameters are measured except for $\mathbf{K_0}$ that is calculated by the Voigt-Reuss-Hill average of all the mineral components.

Sample No.	Sandstone (S1)
Length- L (mm)	72
Bulk modulus of solid grains- K_0(GPa)	45
Porosity- Φ(%)	22.8%
Permeability- κ (mD)	200
Dry bulk modulus at15 MPa-K_d(GPa)	10.8
Dry shear modulus at 15 MPa-G_d(GPa)	7.7
Oil saturation	70%
Brine saturation	30%
Length of the patchy size-ι (mm)	6

Table S2 The fluid properties in our experiment at the pressure of 0.1 MPa and 23 °C. $\mathbf{K_f}$ is the fluid bulk modulus.

Fluid	Viscosity (cP)	Density (kg/cm3)	K_f (GPa)	Compressibility (GPa-1)
Brine	1.08	1100	2.5	0.4
oil	68	890	1.8	0.56

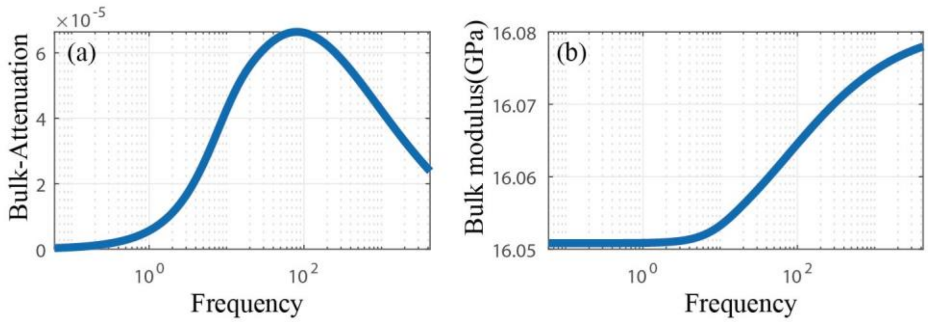

Figure S3. The bulk modulus (a) and attenuation (b) caused by patchy theory. Here the fluid is the mixture of oil (70%) with brine (30%). The patchy size is set as 6 mm.

References :

Borgomano, J. V. M., Pimienta, L., Fortin, J., & Guéguen, Y. (2017). Dispersion and attenuation measurements of the elastic moduli of a dual-porosity limestone. Journal of Geophysical Research: Solid Earth, 122(4), 2016JB013816. https://doi.org/10.1002/2016JB013816

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